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A PROGRAMME FOR THE SOLUTION OF THE MONOENERGETICAL TRANSPORT EQUATION IN THE P5 APPROXIMATION FOR A MULTIREGION CYLINDRICAL GEOMETRY

by

L. MASSIMO

1965

Joint Nuclear Research Center
Ispra Establishment - Italy

Reactor Physics Department
Reactor Theory and Analysis

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Contents

SPHERICAL HARMONICS P5 PROGRAMME [+]

1. General description

The code described in this report was written by the author while at the Dragon Project, A.E.E. Winfrith, for the Ferranti Mercury computer.

Because of the difficulty in the use of double precision matrix operations on that computer the code did not work properly there for many difficult cases.
Now the programme has been translated into Fortran II for the IBM 7090 computer and it works satisfactorily.

The programme solves the monoenergetical transport equation in the P5 approximation for a multiregion cylindrical geometry. Up to 10 concentrical regions can be considered.
The scattering is considered to be isotropic.

Two boundary conditions, reflective and infinite medium, are available, and an air gap may be considered. The equations are solved by the propagation method described for the P_3 approximation in ref. 1.

Such methods have the advantage of solving large regions without losing accuracy in cases where numerical techniques might fail, and are quick where large uniform regions are considered.

While the programming of the P_3 did not present great numerical difficulties, the P_5 required double precision and a tightening up of the procedure of matrix propagation.

Being all analytical the code is very fast (a typical case can run in 15 to 20 seconds) but the numerical difficulties encontered in writing it show that the P_5 is about the highest approximation which one can treat with this method.

As a control on the accuracy of the results, the flux at

[+] Manuscript received on October 5, 1965

each boundary is printed twice with 8 decimal figures, once calculated from the inner side and the other time from the outer side of the boundary. Any difference between the two fluxes is due to numerical inaccuracy.

2. Theory and Code

The expansion of the flux is given by

$$2\pi\psi(\underline{r}, \underline{\Omega}) =$$

$$\frac{1}{2}\psi_{oo}(r) + \frac{3}{2}\psi_{11}(r)\, P_1^1(\eta)\cos\omega + 5\left[\frac{1}{2} P_2(\eta)\,\psi_{o2}(r) + \right.$$

$$\left. + \frac{1}{4!} P_2^2(\eta)\cos 2\omega\,\psi_{22}(r)\right] + 7\left[\frac{2}{4!} P_3^1(\eta)\cos\omega\,\psi_{13}(r) + \right.$$

$$\left. + \frac{1}{6!} P_3^3(\eta)\cos 3\omega\,\psi_{33}(r)\right] +$$

$$+ 9\left[\frac{1}{2} P_4(\eta)\,\psi_{o4}(r) + \frac{2}{6!} P_4^2(\eta)\cos 2\omega\,\psi_{24}(r) + \right.$$

$$\left. + \frac{1}{8!} P_4^4(\eta)\cos 4\omega\,\psi_{44}(r)\right] \quad 11\left[\frac{4!}{6!} P_5^1(\eta)\cos\omega\,\psi_{15}(r) + \right.$$

$$\left. + \frac{2!}{8!} P_5^3(\eta)\cos 3\omega\,\psi_{35}(r) + \frac{1}{10!} P_5^5(\eta)\cos 5\omega\,\psi_{55}(r)\right] \qquad \text{...........} \; (1)$$

and the expressions for the various moments ψ_{mn} in the P5 approximation are as follows:

$$\psi_{oo}(r) = \sum_{i=1}^{3} \{B_i\, I_o(\gamma_i r) + C_i\, K_o(\gamma_i r)\}$$

$$\psi_{11}(r) = \sum_i G_1(\gamma_i)\{B_i I_1(\gamma_i r) - C_i K_1(\gamma_i r)\}$$

$$\psi_{02}(r) = \sum_i -\frac{1}{2} G_2(\gamma_i)\{B_i I_o(\gamma_i r) + C_i K_o(\gamma_i r)\} +$$

$$+ \sum_{j=1}^{2} \{M_j I_o(\beta_j r) + N_j K_o(\beta_j r)\}$$

$$\psi_{22}(r) = \sum_i 3\, G_2(\gamma_i)\{B_i I_2(\gamma_i r) + C_i K_2(\gamma_i r)\} +$$

$$+ 2 \sum_j \{M_j I_2(\beta_j r) + N_j K_2(\beta_j r)\}$$

$$\psi_{13}(r) = \sum_i -\frac{3}{2} G_3(\gamma_i)\{B_i I_1(\gamma_i r) - C_i K_1(\gamma_i r)\} -$$

$$- 5 \sum_j \frac{1}{\beta_j}\{M_j I_1(\beta_j r) - N_j K_1(\beta_j r)\}$$

$$\psi_{33}(r) = \sum_i 15\, G_3(\gamma_i)\{B_i I_3(\gamma_i r) - C_i K_3(\gamma_i r)\} -$$

$$- 30 \sum_j \frac{1}{\beta_j}\{M_j I_3(\beta_j r) - N_j K_3(\beta_j r)\}$$

$$\psi_{04}(r) = \sum_i \frac{3}{8} G_4(\gamma_i)\{B_i I_o(\gamma_i r) + C_i K_o(\gamma_i r)\} +$$

$$+ \sum_j \frac{5(\beta_j^2 - 7)}{12\,\beta_j^2}\{M_j I_o(\beta_j r) + N_j K_o(\beta_j r)\} + Q\, I_o(\nu r) + R\, K_o(\nu r)$$

$$\psi_{24}(r) = \sum_i -\frac{15}{2} G_4(\gamma_i)\{B_i I_2(\gamma_i r) + C_i K_2(\gamma_i r)\} - $$
$$- \sum_j \frac{5(\beta_j^2 - 7)}{\beta_j^2} \{M_j I_2(\beta_j r) + N_j K_2(\beta_j r)\} + 12\{Q I_2(\upsilon r) +$$
$$+ R K_2(\upsilon r)\}$$

$$\psi_{44}(r) = \sum_i 105 G_4(\gamma_i)\{B_i I_4(\gamma_i r) + C_i K_4(\gamma_i r)\} -$$
$$- \sum_j \frac{70(\beta_j^2 - 7)}{\beta_j^2} \{M_j I_4(\beta_j r) + N_j K_4(\beta_j r)\} + 24\{Q I_4(\upsilon r) +$$
$$+ R K_4(\upsilon r)\}$$

$$\psi_{15}(r) = \sum_i \frac{15}{8} G_5(\gamma_i)\{B_i I_1(\gamma_i r) - C_i K_1(\gamma_i r)\} -$$
$$- \sum_j \frac{35(\beta_j^2 - 7)}{44\beta_j} \{M_j I_1(\beta_j r) - N_j K_1(\beta_j r)\} - \frac{9}{\upsilon}\{Q I_1(\upsilon r) -$$
$$- R K_1(\upsilon r)\}$$

$$\psi_{35}(r) = \sum_i -\frac{105}{2} G_5(\gamma_i)\{B_i I_3(\gamma_i r) - C_i K_3(\gamma_i r)\} +$$
$$+ \sum_j \frac{105}{11} \frac{(\beta_j^2 - 7)}{\beta_j} \{M_j I_3(\beta_j r) - N_j K_3(\beta_j r)\} - \frac{324}{\upsilon}\{Q I_3(\upsilon r) - R K_3(\upsilon r)\}$$

$$\psi_{55}(r) = \sum_i 945\, G_5(\gamma_i) \{ B_i\, I_5(\gamma_i r) - C_i\, K_5(\gamma_i r) \} +$$

$$+ \sum_j \frac{3150}{11} \frac{(\beta_j^2 - 7)}{\beta_j} \{ M_j I_5(\beta_j r) - N_j K_5(\beta_j r) \} - \frac{1080}{\nu} \{ Q\, I_5(\nu r) -$$

$$- R\, K_5(\nu r) \} \qquad \text{...........(2a)}$$

where

$G_o = 1$ $\qquad\qquad \alpha = 1 - c$

$G_1 = \frac{\alpha}{\gamma}$ $\qquad\qquad c$ = N° of secondaries per collision

$$G_{n+1} = - \frac{1}{n+1} \left\{ \frac{2n+1}{\gamma} G_n + n\, G_{n-1} \right\} \qquad \text{...........(2b)}$$

and γ_1, γ_2, γ_3 are the positive roots of

$$25\gamma^6 - 21(14 + 11\alpha)\gamma^4 + 35(11 + 34\alpha)\gamma^2 - 1155\alpha = 0$$

(see appendix 2)

β_1 and β_2 are the positive roots of

$$\beta^4 - 18\beta^2 + 33 = 0$$

and $\nu = \sqrt{11}$ $\qquad$ ....(2c)

We can consider the flux as a vector whose components are given by the moments ψ_{mn} so that at a point r in a given region i, it may be represented by

$$\underline{\psi}_i^r = A_i^r\, \underline{C}_i + \underline{S}_i \qquad (3)$$

A_i^r is a (12 x 12) matrix of modified Bessel functions whose arguments are function of the region number i and position r.

$\underline{S}_i^r$ is the source vector (as the source is supposed to be isotropic all the components are zero but the first one).

$\underline{C}_i$ is the vector of the 12 integration constants [B_1, C_1, B_2, ...Q, R] to be determined from the boundary conditions imposed.

If ν is the number of regions and the boundaries are r_1, r_2, ...r_n, the interface conditions of continuity will lead to the following set of (n - 1) equations:

$$A_1^{r_1}\ \underline{C}_1 + \underline{S}_1 = A_2^{r_1}\ \underline{C}_2 + \underline{S}_2$$

$$A_2^{r_2}\ \underline{C}_2 + \underline{S}_2 = A_3^{r_2}\ \underline{C}_3 + \underline{S}_3$$

$$A_{n-1}^{r_{n-1}}\ \underline{C}_{n-1}\ \underline{S}_{n-1} = A_n^{r_{n-1}}\ \underline{C}_n + \underline{S}_n$$

by defining $B_i = (A_{i+1}^{r_i})^{-1}$ we obtain the recurrent formula

$$\underline{C}_{i+1} = B_i(A_i^{r_i}\ \underline{C}_i + \underline{S}_i - \underline{S}_{i+1}) \tag{4}$$

which propagates the coefficients $\underline{C}_i$ outwards, region by region.

The coefficients $\underline{C}_i$ are found by applying the boundary conditions of finiteness of the flux in the centre (the coefficients associated with the K functions must be zero) and the chosen boundary conditions at the last region. If the last region is infinite, in $\underline{C}_n$ the coefficients associated with the I functions must vanish.

If the last boundary is a reflective boundary the components ψ_{11} ; ψ_{13} ; ψ_{33} ; ψ_{15} ; ψ_{35} ; ψ_{55} must vanish [2]. In this latter case the condition will not be applied to $\underline{C}_n$ but to $(A_n^{r_n}\ \underline{C}_n + \underline{S}_n)$.

In both cases we start from the centre with a starter $\underline{C}_1$(12 x 1) with 6 zero components and we must determine the other 6 in such a way that the (12 x 1) response $\underline{R}$ that we get after propagation has zeros in 6 defined positions.

The response $\underline{R}$ will be the sum of a homogeneous response $\underline{R}^*$ (that we would get with all $\underline{S}_i = 0$) and an effect $\underline{b}$ of the sources (that we can get by propagating a $\underline{C}_1$ of all zeros).

$$\underline{R} = \underline{R}^* + \underline{b}$$

If we propagate a matrix C of 6 independent starters (each satisfying to the centre boundary condition) as for example:

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{vmatrix}$$

with all $\underline{S}_i = 0$.
We get an homogeneous response R*.

If $\underline{\phi}$ is a (6 x 1) vector we will have that

$$C_1 \underline{\phi} \longrightarrow R^* \underline{\phi} + \underline{b}$$

We need to find $\underline{\phi}$ such that $R^* \underline{\phi} + \underline{b}$ satisfies the outer boundary condition, and $C_1 \underline{\phi}$ will be the starter $\underline{C}_1$ that solves our problem.

Because of loss of accuracy it is necessary to tighten up the process of obtainment of R*.

To do so we select from R* (which is a 12 x 6 matrix) the six rows which are of interest for imposing the boundary conditions (i.e. the rows corresponding to the coefficients of the

I functions in case of infinite medium, or the rows corresponding to the 6 components of the flux which must vanish in the case of reflective boundary conditions).

Let us call Q this 6 x 6 matrix. If we propagate as a new starter C'_1 the product $C_1\ Q^{-1}$ we will obtain a new R*, out of which we can select a new Q_1. This new Q_1 should be a unit matrix if there would be no loss of accuracy in the process. If Q_1 is not near enough to a unit matrix the loop is continued repropagating $C'_1\ Q_1^{-1}$ until a good enough unit matrix is obtained.

At the end we obtain a starter which gives a response R* which is a unit matrix on the rows of interest for the boundary conditions.

At this point the vector $\underline{\phi}$ is easily determined because $R^*\underline{\phi}$ will be a (12 x 1) vector having the components of $\underline{\phi}$ on the positions of interest for the boundary conditions.

Because we want the 12 x 1 vector $R^*\underline{\phi} + \underline{b}$ to have zeros on those positions, the components of $\underline{\phi}$ will be equal and of opposite sign of the components of $\underline{b}$ corresponding to those positions.

For example in the case of infinite medium, calling ϕ_1, $\phi_2 \cdots\cdots \phi_6$ the components of $\underline{\phi}$ we have

$$R^* = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 1 & 0 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 1 & 0 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 1 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 1 & 0 \\ \hdashline 0 & 0 & 0 & 0 & 0 & 1 \\ \hdashline \end{vmatrix}$$

so that

$$R^*\phi = \begin{vmatrix} \phi_1 \\ \text{--} \\ \phi_2 \\ \text{--} \\ \phi_3 \\ \text{---} \\ \phi_4 \\ \text{---} \\ \phi_5 \\ \text{---} \\ \phi_6 \\ \text{---} \end{vmatrix}$$

and to satisfy the boundary conditions we will need:

$$\phi_1 = - b_1$$
$$\phi_2 = - b_3$$
$$\phi_3 = - b_5$$
$$\phi_4 = - b_7$$
$$\phi_5 = - b_9$$
$$\phi_6 = - b_{11}$$

At this point the programme can calculate the coefficients $\underline{C}_i$ in every region. If we call C^*_i the (12x6) matrices that we got as intermediate results while calculating R*, and $\underline{b}_i$ the corresponding intermediate results obtained while calculating $\underline{b}$, we have:

$$\underline{C}_i = C^*_i \, \underline{\phi} + \underline{b}_i$$

The flux is then calculated in the requested points.

The output also contains the mean flux in every region and an asymptotic flux and current for extrapolation lengths calculations.

3. References

1. J.R.Askew, R.J.Brissenden
Mercury programme 560 - Spherical harmonics P_3 approximation - One group, cylindrical geometry.
AEEW-M116, January 1961

2. J.Tait
The calculation of the fine structure of the thermal neutron flux in a pile, by the sphèrical harmonics method.
A/CONF. 8/P/433 - 1955

4. Acknowledgment

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The translation into Fortran is largely due to Mr. G.Fassone.

Appendix 1

The air-gap condition

The equations connecting the various moments across the air gap are as follows:

$$a\psi_{11}(a) = c\psi_{11}(c) \qquad (5.1)$$

$$a\psi_{13}(a) = c\psi_{13}(c) \qquad (5.2)$$

$$a\psi_{15}(a) = c\psi_{15}(c) \qquad (5.3)$$

$$a^3\{\psi_{33}(a) - 12\psi_{11}(a) + 2\psi_{13}(a)\} = c^3\{\psi_{33}(c) - 12\psi_{11}(c) + 2\psi_{13}(c)\} \qquad \ldots\ldots (5.4)$$

$$a^3\{\psi_{35}(a) - 12\psi_{11}(a) - 28\psi_{13}(a) + 12\psi_{15}(a)\} = c^3\{\psi_{35}(c) - 12\psi_{11}(c) - 28\psi_{13}(c) + 12\psi_{15}(c)\} \qquad \ldots\ldots (5.5)$$

$$a^5\{\psi_{55}(a) + 1296\psi_{11}(a) - 336\psi_{13}(a) + 48\psi_{15}(a) - 168\psi_{33}(a) + 6\psi_{35}(a)\} = c^5\{\psi_{55}(c) + 1296\psi_{11}(c) - 336\psi_{13}(c) + 48\psi_{15}(c) - 168\psi_{33}(c) + 6\psi_{35}(c)\} \qquad \ldots\ldots (5.6)$$

$$
\begin{aligned}
&-\frac{\pi}{4}\psi_{00}(a) + \frac{\pi}{2}\psi_{11}(a) + \frac{5\pi}{32}\psi_{02}(a) - \frac{5\pi}{64}\psi_{22}(a) + \frac{9\pi}{256}\psi_{04}(a) - \\
&- \frac{\pi}{256}\psi_{24}(a) + \frac{\pi}{2048}\psi_{44}(a) = \\
&- \frac{\pi}{4}\psi_{00}(c) + \frac{c}{a}\left\{\sin^{-1}\frac{a}{c} + \frac{a}{c}\sqrt{1-\frac{a^2}{c^2}}\right\}\psi_{11}(c) + \frac{5\pi}{32}\psi_{02}(c) - \\
&- \frac{15\pi}{64}\left(1 - \frac{2}{3}\frac{a^2}{c^2}\right)\psi_{22}(c) + \frac{7}{45}\left(1 - \frac{a^2}{c^2}\right)^{3/2}\psi_{33}(c) + \frac{9\pi}{256}\psi_{04}(c) - \\
&- \frac{3\pi}{256}\left(1 - \frac{2}{3}\frac{a^2}{c^2}\right)\psi_{24}(c) - \frac{\pi}{2048}\left(15 - 40\frac{a^2}{c^2} + 24\frac{a^4}{c^4}\right)\psi_{44}(c) + \\
&+ \frac{11}{1260}\left(1 - \frac{a^2}{c^2}\right)^{3/2}\psi_{35}(c) + \frac{11}{4200}\left(1 - \frac{a^2}{c^2}\right)^{3/2}\left(1 - \frac{8}{3}\frac{a^2}{c^2}\right)\psi_{55}(c)
\end{aligned}
\qquad \ldots\ldots (5.7)
$$

$$-\frac{3\pi}{32}\psi_{00}(a) - \frac{105\pi}{128}\psi_{02}(a) + \frac{5\pi}{256}\psi_{22}(a) + \frac{\pi}{2}\psi_{13}(a) +$$

$$+ \frac{2079\pi}{4096}\psi_{04}(a) - \frac{171\pi}{4096}\psi_{24}(a) - \frac{9\pi}{32768}\psi_{44}(a) = -\frac{3\pi}{32}\psi_{00}(c) -$$

$$- \frac{105\pi}{128}\psi_{02}(c) + \frac{15\pi}{256}\left(1 - \frac{2}{3}\frac{a^2}{c^2}\right)\psi_{22}(c) + \frac{c}{a}\left\{\sin^{-1}\frac{a}{c} +\right.$$

$$\left. + \frac{a}{c}\sqrt{1 - \frac{a^2}{c^2}}\right\}\psi_{13}(c) - \frac{1}{15}\left(1 - \frac{a^2}{c^2}\right)^{3/2}\psi_{33}(c) + \frac{2079\pi}{4096}\psi_{04}(c) -$$

$$- \frac{513\pi}{4096}\left(1 - \frac{2}{3}\frac{a^2}{c^2}\right)\psi_{24}(c) + \frac{9\pi}{32768}\left(15 - 40\frac{a^2}{c^2} + 24\frac{a^4}{c^4}\right)\psi_{44}(c) +$$

$$+ \frac{11}{210}\left(1 - \frac{a^2}{c^2}\right)^{3/2}\psi_{35}(c) - \frac{11}{6300}\left(1 - \frac{a^2}{c^2}\right)^{3/2}\left(1 - \frac{8}{3}\frac{a^2}{c^2}\right)\psi_{55}(c)$$

......(5.8)

$$-\frac{15\pi}{256}\psi_{00}(a) - \frac{1425\pi}{4096}\psi_{02}(c) + \frac{25\pi}{8192}\psi_{22}(a) - \frac{42255\pi}{32768}\psi_{04}(a) +$$

$$+ \frac{495\pi}{32768}\psi_{24}(a) + \frac{9\pi}{262144}\psi_{44}(a) + \frac{\pi}{2}\psi_{15}(a) = -\frac{15\pi}{256}\psi_{00}(c) -$$

$$- \frac{1425\pi}{4096}\psi_{02}(c) + \frac{75\pi}{8192}\left(1 - \frac{2}{3}\frac{a^2}{c^2}\right)\psi_{22}(c) - \frac{42255\pi}{32768}\psi_{04}(c) +$$

$$+ \frac{1485\pi}{32768}\left(1 - \frac{2}{3}\frac{a^2}{c^2}\right)\psi_{24}(c) - \frac{7\pi}{262144}\left(15 - 40\frac{a^2}{c^2} + 24\frac{a^4}{c^4}\right)\psi_{44}(c) +$$

$$+ \frac{c}{a}\left\{\sin^{-1}\frac{a}{c} + \frac{a}{c}\sqrt{1 - \frac{a^2}{c^2}}\right\}\psi_{15}(c) - \frac{1}{28}\left(1 - \frac{a^2}{c^2}\right)^{3/2}\psi_{35}(c) +$$

$$+ \frac{1}{2520}\left(1 - \frac{a^2}{c^2}\right)^{3/2}\left(1 - \frac{8}{3}\frac{a^2}{c^2}\right)\psi_{55}(c)$$

......(5.9)

$$\frac{15\psi}{16}\psi_{00}(a) - \frac{75\pi}{64}\psi_{02}(a) - \frac{225\pi}{128}\psi_{22}(a) + \frac{\pi}{2}\psi_{33}(a) + \frac{405\pi}{2048}\psi_{04}(a) +$$

$$+ \frac{135\pi}{2048}\psi_{24}(a) - \frac{675\pi}{16384}\psi_{44}(a) = \frac{15\pi}{16}\psi_{00}(c) + 12\frac{c}{a}\left(1 - \frac{c^2}{a^2}\right)$$

$$\left\{\sin^{-1}\frac{a}{c} - \frac{a}{c}\sqrt{1 - \frac{a^2}{c^2}}\right\}\psi_{11}(c) - \frac{75\pi}{64}\psi_{02}(c) +$$

$$+ \frac{375\pi}{128} \left(\frac{1}{3} - \frac{14}{15} \frac{a^2}{c^2}\right)\psi_{22}(c) - 2 \frac{c}{a}\left(1- \frac{c^2}{a^2}\right)\left\{\sin^{-1} \frac{a}{c} - \frac{a}{c}\sqrt{1- \frac{a^2}{c^2}}\right\}\psi_{13}(c) +$$

$$+ \frac{c^3}{a^3}\left\{\sin^{-1} \frac{a}{c} - \frac{a}{c}\sqrt{1 - \frac{a^2}{c^2}} \left(1 + \frac{4}{3} \frac{a^2}{c^2} - \frac{10}{3} \frac{a^4}{c^4}\right)\right\} \psi_{33}(c) +$$

$$+ \frac{405\pi}{2048} \psi_{04} (c) - \frac{225\pi}{2048} \left(\frac{1}{3} - \frac{14}{15} \frac{a^2}{c^2}\right) \psi_{24} (c) +$$

$$+ \frac{15\pi}{16384} \left(35 - 392 \frac{a^2}{c^2} + 312 \frac{a^4}{c^4}\right) \psi_{44}(c) - \frac{11}{945} \left(1 - \frac{a^2}{c^2}\right)^{\frac{1}{2}}$$

$$\left(1 - 17 \frac{a^2}{c^2} + 16 \frac{a^4}{c^4}\right) \psi_{55}(c) \qquad \ldots\ldots(5.10)$$

$$\frac{105\pi}{64} \psi_{00}(a) + \frac{5775\pi}{1024} \psi_{02}(a) - \frac{1575\pi}{2048} \psi_{22}(a) - \frac{82215\pi}{8192} \psi_{04}(a) -$$

$$- \frac{20601\pi}{8192} \psi_{24}(a) + \frac{945\pi}{32768} \psi_{44}(a) + \frac{\pi}{2} \psi_{35}(a) = \frac{105\pi}{64} \psi_{00}(c) +$$

$$+ 12 \frac{c}{a} \left(1 - \frac{c^2}{a^2}\right) \left\{ \sin^{-1} \frac{a}{c} - \frac{a}{c} \sqrt{1 - \frac{a^2}{c^2}}\right\} \psi_{11}(c) + \frac{5775\pi}{1024} \psi_{02}(c) +$$

$$+ \frac{175\pi}{2048} \left(5 - 14 \frac{a^2}{c^2}\right)\psi_{22}(c) + 28 \frac{c}{a} \left(1 - \frac{c^2}{a^2}\right) \left\{\sin^{-1} \frac{a}{c} -\right.$$

$$\left.- \frac{a}{c}\sqrt{1- \frac{a^2}{c^2}}\right\} \psi_{13}(c) - \frac{82215\pi}{8192} \psi_{04}(c)+ \frac{2289\pi}{8192} \left(5-14 \frac{a^2}{c^2}\right) \psi_{24}(c) -$$

$$- \frac{21\pi}{65536} \left(35 - 390 \frac{a^2}{c^2} + 312 \frac{a^4}{c^4}\right)\psi_{44}(c) -12 \frac{c}{a}\left(1- \frac{c^2}{a^2}\right) \left\{\sin^{-1} \frac{a}{c} -\right.$$

$$\left.- \frac{a}{c}\sqrt{1 - \frac{a^2}{c^2}}\right\} \psi_{15}(c)+ \frac{c^3}{a^3}\left\{\sin^{-1} \frac{a}{c} - \frac{a}{c}\sqrt{1- \frac{a^2}{c^2}} \left(1+ \frac{4}{3} \frac{a^2}{c^2} - \frac{10}{3} \frac{a^4}{c^4}\right)\right\}\psi_{35}(c)+$$

$$+ \frac{1}{135} \left(1 - \frac{a^2}{c^2}\right)^{\frac{1}{2}} \left(1 - 17 \frac{a^2}{c^2} + 16 \frac{a^4}{c^4}\right) \psi_{55}(c) \qquad \ldots\ldots(5.11)$$

$$- \frac{945\pi}{32}\psi_{00}(a) + \frac{23625\pi}{512}\psi_{02}(a)+ \frac{39375\pi}{1024}\psi_{22}(a)- \frac{76545\pi}{4096}\psi_{04}(a)- \frac{14175\pi}{4096}\psi_{24}$$

$$(a) - \frac{99225\pi}{32768}\psi_{44}(a) + \frac{\pi}{2} \psi_{55}(a) = - \frac{945\pi}{32}\psi_{00}(c) + 648 \frac{c^5}{a^5} \left\{\left(2-3\frac{a^2}{c^2} +\right.\right.$$

$$+ \frac{a^4}{c^4}) \sin^{-1}\frac{a}{c} - \frac{a}{c}(1 - \frac{a^2}{c^2})^{\frac{1}{2}} (2 - \frac{5}{3}\frac{a^2}{c^2} - \frac{1}{3}\frac{a^4}{c^4})\} \psi_{11}(c) +$$

$$+ \frac{23625\pi}{512}\psi_{02}(c) - \frac{1575\pi}{1024}(21 - 46\frac{a^2}{c^2})\psi_{22}(c) - 168\frac{c^5}{a^5}\{(2 -$$

$$- 3\frac{a^2}{c^2} + \frac{a^4}{c^4}) \sin^{-1}\frac{a}{c} - \frac{a}{c}(1 - \frac{a^2}{c^2})^{\frac{1}{2}} (2 - \frac{5}{3}\frac{a^2}{c^2} - \frac{1}{3}\frac{a^4}{c^4})\}\psi_{13}(c) -$$

$$- 56\frac{c^5}{a^5}\{3(1 - \frac{a^2}{c^2}) \sin^{-1}\frac{a}{c} - \frac{a}{c}(1 - \frac{a^2}{c^2})^{\frac{1}{2}} (3 - \frac{a^2}{c^2} - 2\frac{a^6}{c^6})\}\psi_{33}(c) -$$

$$- \frac{76545\pi}{4096}\psi_{04}(c) + \frac{567\pi}{4096}(21 - 46\frac{a^2}{c^2})\psi_{24}(c) - \frac{567\pi}{32768}(63 - 552\frac{a^2}{c^2} + 664\frac{a^4}{c^4})\psi_{44}$$

$$+ 24\frac{c^5}{a^5}\{(2 - 3\frac{a^2}{c^2} + \frac{a^4}{c^4}) \sin^{-1}\frac{a}{c} - \frac{a}{c}(1 - \frac{a^2}{c^2})^{\frac{1}{2}} (2 - \frac{5}{3}\frac{a^2}{c^2} - \frac{1}{3}\frac{a^4}{c^4})\}\psi_{15}(c) +$$

$$+ 2\frac{c^5}{a^5}\{3(1 - \frac{a^2}{c^2}) \sin^{-1}\frac{a}{c} - \frac{a}{c}\sqrt{1 - \frac{a^2}{c^2}} (3 - \frac{a^2}{c^2} - 2\frac{a^6}{c^6})\}\psi_{35}(c) +$$

$$+ \frac{c^5}{a^5}\{\sin^{-1}\frac{a}{c} - \frac{a}{c}\sqrt{1 - \frac{a^2}{c^2}} (1 + \frac{2}{3}\frac{a^2}{c^2} + \frac{2}{15}\frac{a^4}{c^4} + \frac{86}{15}\frac{a^6}{c^6} - \frac{128}{15}\frac{a^8}{c^8})\}\psi_{55}(c) \qquad \ldots\ldots (5.12)$$

where a and c are the inner and outer radii of the air gap.

Those equations can be represented by the matrix equation:

$$X(A_{j-1}^{r_{j-1}}\,\underline{C}_{j-1} + \underline{S}_{j-1}) = Y(A_{j+1}^{r_j}\,\underline{C}_{j+1} + \underline{S}_{j+1}) \qquad \ldots\ldots (6)$$

where the region j is considered to be the air gap. It is convenient to include the air gap conditions in the chain (4) as part of the propagation [Ref. 1]. For this purpose we must find fictitious $A_j^{r_{j-1}}$, $A_j^{r_j}$, $\underline{S}_j$ such that the relation

$$A_{j-1}^{r_{j-1}}\,\underline{C}_{j-1} + \underline{S}_{j-1} = A_j^{r_{j-1}}\,\underline{C}_j + \underline{S}_j$$

$$A_j^{r_j}\,\underline{C}_j + \underline{S}_j = A_{j+1}^{r_j}\,C_{j+1} + \underline{S}_{j+1}$$

is equivalent to the (6). For that we need:

$$\underline{S}_j = 0 \qquad A_j^{r_{j-1}} = X^{-1} \qquad A_j^{r_j} = Y^{-1}$$

Appendix 2

Numerical calculations of the $G(\gamma)$ functions

The formulae given for the $G(\gamma)$ functions are not suitable for numerical calculations, because, for very small values of α , $\gamma^2 \approx 3\alpha$, and as it can easily be seen, in the equation (2b) the term $3\alpha - \gamma^2$ appears.

From the bicubic we can get

$$(3\alpha - \gamma^2) = \frac{1}{385} \left[25 \gamma^6 - 21 (14 + 11\alpha) \gamma^4 + 1190 \alpha\gamma^2 \right]$$

Having made this substitution we get:

$$G_1 (\gamma) = - \frac{\alpha}{\gamma}$$

$$G_2 (\gamma) = 0.0324675324 \gamma^4 - 0.381818182 \gamma^2 - 0.3 \gamma^2\alpha + 1.5454545$$

$$G_3 (\gamma) = - 0.041322314 \gamma^5 + 0.381818182 \alpha\gamma^3 + 0.431837856 \gamma^3 - 1.4669421$$

$$G_4 (\gamma) = 0.03047744 \gamma^6 - 0.31045134 \gamma^4 - 0.28161156 \alpha\gamma^4 + 1.007544412 \alpha\gamma$$

$$G_5 (\gamma) = -0.0138533847 \gamma^7 + 0.1411142654 \gamma^5 + 0.128005276 \alpha\gamma^5 - 0.457974$$

Appendix 3

Input description

Card	Format	Name	Description
1	E 12.5	EPS1	Accuracy on γ coefficients
	E 12.5	EPS2	Accuracy on unit matrix
	I 12	NINV1	N° of iterations in 12x12 matrix version
	I 12	NINV2	N° of iterations in 6x6 matrix in sion
	I12	IMP	= 0 macroscopic input = N microscopic input with a libr of N materials

If IMP = N then N card 2 are needed

Card	Format	Name	Description
2	E 12.5	TRANSP(I)	Microscopic transport cross secti material I
	E 12.5	ABS(I)	Microscopic absorption cross sect material I
	A 12	TITLE(I)	Name of material I
3	I 12	NREGS	N° of regions ($\leqslant$ 10)
	I 12	NX	= 0 no effect = 1 print intermediate data (normally not necessary)

If IMP > 0 then cards 4 and 5 are needed for each region

Card	Format	Name	Description
4	E 12.5	RAD(IN)	Outer radius region IN
	E 12.5	SOURCE(IN)	Source (neutrons/unit volume) region IN
	E 12.5	AMAT	N° of materials in region IN
	E 12.5	APRIN(IN)	N° of intermediate print-out poin region IN

Each card 4 is followed by AMAT card 5.

Card	Format	Name	Description
5	I 12	NMAT	Material N° (given by the order in which the material appears i card 2)
	E 12.5	CONC	Concentration

If IMP = 0 then card 6 is needed for each region

Card	Format	Name	Description
6	E 12.5	RAD(I)	Outer radius region I
	E 12.5	FMP(I)	Mean free path region I (negati means air gap)
	E 12.5	SOURCE(I)	Source (neutrons/unit volume) r gion I
	E 12.5	ALPHA(I)	N° absorptions/collision regior
	E 12.5	blank	
	E 12.5	APRIN(I)	N° of intermediate print-out pe region I
7	I 12	IQ	= 0 infinite medium = 1 reflective boundary.

```
      EPS1              ACCURACY ON GAMMA COEFFICIENTS
      EPS2              ACCURACY ON UNIT MATRIX
      NINV1             NO. OF ITERATIONS IN 12*12 MATRIX INVERSION
      NINV2             NO. OF ITERATIONS IN 6*6 MATRIX INVERSION
      NREGS             NO. OF REGIONS
      NX                =0 NO EFFECT,      =1 PRINT INTERMEDIATE DATA
      RAD(IN)           OUTER RADIUS REGION IN
      FMP(IN)           MEAN FREE PATH REGION IN
                        A NEGATIVE MEAN FREE PATH MEANS AIR GAP
      SOURCE(IN)        SOURCE (NEUTRONS/UNIT VOLUME) REGION IN
      ALPHA(IN)         NO. ABSORPTIONS/COLLISION REGION IN
      AMASS(IN)         ATOMIC WEIGHT OF SCATTERER REGION IN(NOT USED NOW)
      APRIN(IN)         NUMBER OF INTERMEDIATE PRINT OUT POINTS REGION IN
      IMP               =0 MACROSCOPIC INPUT
                        =N MICROSCOPIC INPUT WITH A LIBRARY OF N MATERIALS
      TRANSP(I)         MICROSCOPIC LIBRARY  SIGMA TRANSP. MATERIAL  I
      ABS(I)            MICROSCOPIC LIBRARY  SIGMA ABSORPTION MAT. I
      TITLE(I)          MICROSCOPIC LIBRARY  NAME MATERIAL I
      CONC              CONCENTRATION
      AMAT              NO. OF MATERIALS REGION IN
      NMAT              MATERIAL NUMBER (IN LIBRARY)


      DIMENSION         A(12,12,10),    B(12,12,10),     U(12,6,10),
     1 VAN(12,12),      U1(12,6),       RESP(12,12),     V(12,10),
     2 RV(6),           V1(12),         S(12,10),        C(12,10),
     3 RAD(10),FMP(10),SOURCE(10),ALPHA(10),AMASS(10),APRIN(10)
      DIMENSION Z(12,12),Z1(12,12)
      DIMENSION Y(1),RADM(1)
      DIMENSION EPS1(1),EPS2(1)
      DIMENSION TRANSP(50),ABS(50),TITLE(50)
      COMMON A,B,U,VAN,U1,RESP,V,RV,V1,S,C,RAD,FMP,SOURCE,ALPHA,AMASS,A
     1RIN,IR,IRR,IN,IQ,N5,N6
      COMMON Z,Z1,Y,RADM,NREGS
      COMMON NX
      COMMON EPS1,EPS2,NINV1,NINV2
    1 FORMAT (6E12.5)
    2 FORMAT (6I12)
    4 FORMAT(2E12.5,3I12)
 3001 FORMAT(2E12.5,A12)
 3002 FORMAT(1H1,36X,20H MICROSCOPIC LIBRARY/////33X,10H TRANSPORT,11X,
     111H ABSORPTION,11X,5H NAME///)
 3003 FORMAT(E44.5,E21.5,A17)
 3004 FORMAT(120H1   REGION      OUTER RADIUS         SOURCE        NO. MATE
     1IALS          NAME       CONCENTRATION    NO. ABS/COLL    MEAN FREE PATH
 3005 FORMAT(4E12.5)
 3006 FORMAT(I8,E18.5,2E16.5)
 3007 FORMAT (I12,E12.5)
 3008 FORMAT(A71,E16.5)
 3009 FORMAT(E103.5,E15.5)

      TAPE DESIGNATION

      N5=5
```

```
 P5MAIN
        N6=6
C
        READ INPUT TAPE N5,4,EPS1,EPS2,NINV1,NINV2,IMP
        IF(IMP) 999,999,8000
  8000  READ INPUT TAPE N5,8001,(TRANSP(I),ABS(I),TITLE(I),I=1,IMP)
        WRITE OUTPUT TAPE N6,8002
        WRITE OUTPUT TAPE N6,8003,(TRANSP(I),ABS(I),TITLE(I),I=1,IMP)
   999  READ INPUT TAPE N5,2,NREGS,NX
        IF(IMP) 8060,8060,8051
  8051  WRITE OUTPUT TAPE N6,8004
        DO 8101 IN=1,NREGS
        READ INPUT TAPE N5,8005,RAD(IN),SOURCE(IN),AMAT,APRIN(IN)
        WRITE OUTPUT TAPE N6,8006,IN,RAD(IN),SOURCE(IN),AMAT
        MAT=AMAT+0.01
        SIGT=0.0
        SIGA=0.0
        DO 8100 I=1,MAT
        READ INPUT TAPE N5,8007,NMAT,CONC
        WRITE OUTPUT TAPE N6,8008,TITLE(NMAT),CONC
        SIGT=SIGT+CONC*TRANSP(NMAT)
  8100  SIGA=SIGA+CONC*ABS(NMAT)
        ALPHA(IN)=SIGA/SIGT
        FMP(IN)=1.0/SIGT
        WRITE OUTPUT TAPE N6,8009,ALPHA(IN),FMP(IN)
  8101  CONTINUE
        GO TO 8052
  8060  READ INPUT TAPE N5,1,(RAD(I),FMP(I),SOURCE(I),ALPHA(I),AMASS(I
      1 APRIN(I),I=1,NREGS)
        WRITE OUTPUT TAPE N6,3,NREGS
     3  FORMAT(1H1,I5,9H REGIONS //)
        WRITE OUTPUT TAPE N6,5003,(I,RAD(I),FMP(I),SOURCE(I),ALPHA(I),I
      1 NREGS)
  5003  FORMAT(////18X,10H REGION = I3/,12X,16H OUTER RADIUS = E12.5/
      1 10X,18H MEAN FREE PATH = E12.5/,18X,10H SOURCE = E12.5/
      2 28H NO.ABSORPTIONS/COLLISION = E12.5/)
C
C       INFINITE MEDIUM      IQ=0
C       REFLECTIVE BOUNDARY IQ=1
C
  8052  READ INPUT TAPE N5,2,IQ
        DO 20 I=1,NREGS
        IF(ALPHA(I)-0.00001)21,20,20
D   21  ALPHA(I)=0.00001
        WRITE OUTPUT TAPE N6,22,I
    22  FORMAT (8H0REGION I5,19H IS PURE SCATTERER //)
    20  CONTINUE
        NR=NREGS-1
        DO 150 INN=1,NR
        IN=INN
C
C       TEST ON AIR GAP
C
        IF(FMP(IN))6500,6501,6501
  6501  CONTINUE
D       RADM=RAD(IN)/FMP(IN)
D       Y=ALPHA(IN)
```

P5MAIN

```
      CALL MATRIX
      DO 23 I=1,12
      DO 23 J=1,12
   23 A(I,J,IN)=Z(I,J)

      TEST ON AIR GAP

      IF(FMP(IN+1))6502,6500,6500
 6502 CALL AIRGAP
      GO TO 150
 6500 CONTINUE

      RADM=RAD(IN)/FMP(IN+1)
      Y=ALPHA(IN+1)
      CALL MATRIX
      CALL SIMH(Z,Z1,12,NINV1)
      DO 24 I=1,12
      DO 24 J=1,12
   24 B(I,J,IN)=Z1(I,J)
  150 CONTINUE

      DIRECT MATRICES ARE STORED IN A(I,J,IN) FOR EACH RADIUS IN
      INVERTED MATRICES ARE STORED IN B(I,J,IN) FOR EACH RADIUS IN

      IF(IQ)101,50,101
  101 RADM=RAD(NREGS)/FMP(NREGS)
      Y=ALPHA(NREGS)
      CALL MATRIX
      DO 25 I=1,12
      DO 25 J=1,12
   25 A(I,J,NREGS)=Z(I,J)
   50 DO 100 I=1,NREGS
      S(1,I)=SOURCE(I)*FMP(I)/ALPHA(I)
      DO 100 J=2,12
  100 S(J,I)=0.0

      NOW ALL MATRICES ARE PREPARED
      PREPARE FIRST STARTER

      IN=1
      DO 500 I=1,6
      DO 500 J=1,12
  500 U(I,J,IN)=0.0
      J=0
      DO 501 I=1,11,2
      J=J+1
  501 U(I,J,IN)=1.0

      PROPAGATION OF STARTER
      NITER=0
 1001 DO 1000 IN=2,NREGS

      MATRIX MULTIPLICATION   A*U=U1

      DO 502 I=1,12
      DO 502 J=1,6
```

```
PSMAIN
      U1(I,J)=0.0
      DO 502 IS=1,12
  502 U1(I,J)=U1(I,J)+A(I,IS,IN-1)*U(IS,J,IN-1)
C
C     MATRIX MULTIPLICATION   B*U1=U
C
      DO 503 I=1,12
      DO 503 J=1,6
      U(I,J,IN)=0.0
      DO 503 IS=1,12
  503 U(I,J,IN)=U(I,J,IN)+B(I,IS,IN-1)*U1(IS,J)
 1000 CONTINUE
      IF(IQ)504,505,504
C
C     MATRIX MULTIPLICATION   A*U=U1
C
  504 DO 506 I=1,12
      DO 506 J=1,6
      U1(I,J)=0.0
      DO 506 IS=1,12
  506 U1(I,J)=U1(I,J)+A(I,IS,NREGS)*U(IS,J,NREGS)
C
      N=1
      I=1
      DO 510 K=1,3
      I=I+K
      DO 510 KK=1,K
      DO 507 J=1,6
  507 VAN(N,J)=U1(I,J)
      N=N+1
  510 I=I+1
      GO TO 400
  505 N=1
      I=1
      DO 508 K=1,6
      DO 509 J=1,6
  509 VAN(N,J)=U(I,J,NREGS)
      N=N+1
  508 I=I+2
C
C     VAN(I,J) IS A 6*6 MATRIX OBTAINED SELECTING FROM THE RESPONSE
C     6 ROWS OF INTEREST FOR THE OUTER BOUNDARY CONDITIONS
C
C     CHECK IF VAN(I,J) IS UNIT MATRIX
  400 DO 401 I=1,6
      DO 401 J=1,6
      IF(I-J)402,403,402
  403 RESP(I,J)=VAN(I,J)-1.0
      GO TO 401
  402 RESP(I,J)=VAN(I,J)
  401 RESP(I,J)=ABSF(RESP(I,J))
      RMAX=RESP(1,1)
      DO 404 I=1,6
      DO 404 J=1,6
      IF(RMAX-RESP(I,J))405,404,404
  405 RMAX=RESP(I,J)
```

```
 P5MAIN
   404 CONTINUE
C
       IF(NX)5002,5002,5001
  5001 CONTINUE
       DO 5000 KK=1,NREGS
       WRITE OUTPUT TAPE N6,2003,KK
  2003 FORMAT(1H0//,17H U MATRIX REGION I5//)
       WRITE OUTPUT TAPE N6,2000,((U(I,J,KK),J=1,6),I=1,12)
  2000 FORMAT(6E20.5)
  5000 WRITE OUTPUT TAPE N6,2001
  2001 FORMAT(1H0//)
       WRITE OUTPUT TAPE N6,2000,((VAN(I,J),J=1,6),I=1,6)
       WRITE OUTPUT TAPE N6,2004,RMAX
  2004 FORMAT(7H0 RMAX= E12.5//)
       WRITE OUTPUT TAPE N6,2002
  2002 FORMAT(1H1//)
  5002 CONTINUE
C
       NITER=NITER+1
       IF(RMAX-EPS2)409,409,407
   407 IF(NITER-10)408,408,409
D  408 CALL SIMH(VAN,RESP,6,NINV2)
C
C
C      MULTIPLICATION OF OLD STARTER BY THE INVERTED VAN(I,J) AND
C      REPETITION OF THE CYCLE
C
       DO 450 I=1,12
       DO 450 J=1,6
D      U1(I,J)=0.0
       DO 450 IS=1,6
D  450 U1(I,J)=U1(I,J)+U(I,IS,1)*RESP(IS,J)
       DO 451 I=1,12
       DO 451 J=1,6
D  451 U(I,J,1)=U1(I,J)
       GO TO 1001
C
   409 WRITE OUTPUT TAPE N6,410,NITER,RMAX,EPS2
   410 FORMAT(////49H1 MAXIMUM OFF-DIAGONAL TERM OF UNIT MATRIX AFTER
      1 18H ITERATIONS RMAX = E12.5,13H WHILE EPS2 = E12.5///)
C
C      PROPAGATION OF A ZERO STARTER TO GET THE EFFECT OF THE SOURCES
C
   406 DO 550 I=1,12
D  550 V(I,1)=0.0
       DO 1010 IN=2,NREGS
       DO 551 I=1,12
D      V1(I)=0.0
       DO 551 IS=1,12
D  551 V1(I)=V1(I)+A(I,IS,IN-1)*V(IS,IN-1)
       DO 552 I=1,12
D  552 V1(I)=V1(I)+S(I,IN-1)-S(I,IN)
       DO 553 I=1,12
D      V(I,IN)=0.0
       DO 553 IS=1,12
D  553 V(I,IN)=V(I,IN)+B(I,IS,IN-1)*V1(IS)
  1010 CONTINUE
```

P5MAIN

```
      IF(IQ)554,555,554
  554 DO 560 I=1,12
      V1(I)=0.0
      DO 560 IS=1,12
  560 V1(I)=V1(I)+A(I,IS,NREGS)*V(IS,NREGS)
      RV(1)=-V1(2)
      RV(2)=-V1(5)
      RV(3)=-V1(6)
      RV(4)=-V1(10)
      RV(5)=-V1(11)
      RV(6)=-V1(12)
      GO TO 556
  555 RV(1)=-V(1,NREGS)
      RV(2)=-V(3,NREGS)
      RV(3)=-V(5,NREGS)
      RV(4)=-V(7,NREGS)
      RV(5)=-V(9,NREGS)
      RV(6)=-V(11,NREGS)

      CALCULATION OF THE VECTOR C(I,IN) OF THE COEFFICIENTS IN EACH
      REGION IN

  556 DO 600 IN=1,NREGS
      DO 650 I=1,12
      C(I,IN)=0.0
      DO 650 IS=1,6
  650 C(I,IN)=C(I,IN)+U(I,IS,IN)*RV(IS)
      DO 651 I=1,12
  651 C(I,IN)=C(I,IN)+V(I,IN)
  600 CONTINUE

      IF(IQ) 7000,7001,7000
 7001 WRITE OUTPUT TAPE N6,7002
 7002 FORMAT(21H1 CONTROL ON ACCURACY /34H  COEFFICIENTS IN THE LAST
     1ION /47H  THE COEFFICIENTS MARKED WITH * SHOULD VANISH //)
      WRITE OUTPUT TAPE N6,7003,(C(I,NREGS),I=1,12)
 7003 FORMAT(E13.5,2H */E13.5)
 7000 CONTINUE

      IF(IQ)652,653,652
  653 DO 654 I=1,12,2
  654 C(I,NREGS)=0.0
  652 CONTINUE

      WRITE OUTPUT TAPE N6,3000,(RV(I),I=1,6)
 3000 FORMAT(42H  EFFECT OF THE SOURCES ON A ZERO STARTER //6E20.5//
      WRITE OUTPUT TAPE N6,3001
 3001 FORMAT(31H0 COEFFICIENTS FOR EACH REGION //)
      IF(NREGS-6)3002,3002,3003
 3002 DO 3006 I=1,12
 3006 WRITE OUTPUT TAPE N6,2000,(C(I,IN),IN=1,NREGS)
      GO TO 3004
 3003 WRITE OUTPUT TAPE N6,2000,((C(I,IN),IN=1,6),I=1,12)
      J=NREGS-6
      DO 3005 I=1,12
 3005 WRITE OUTPUT TAPE N6,2000,(C(I,IN),IN=1,J)
```

P5MAIN

```
 3004 CONTINUE

      CALL PRINT
      GO TO 999
      END(1,0,0,0,0,0,1,0,0,0,0,0,0,0,0)
```

```
ATRIX
      SUBROUTINE MATRIX
      DIMENSION      A(12,12,10),    B(12,12,10),    U(12,6,10),
     1 VAN(12,12),   U1(12,6),       RESP(12,12),    V(12,10),
     2 RV(6),        V1(12),         S(12,10),       C(12,10),
     3 RAD(10),FMP(10),SOURCE(10),ALPHA(10),AMASS(10),APRIN(10)
      DIMENSION Z(12,12),Z1(12,12)
      DIMENSION Y(1),RADM(1)
      DIMENSION EPS1(1),EPS2(1)
      COMMON A,B,U,VAN,U1,RESP,V,RV,V1,S,C,RAD,FMP,SOURCE,ALPHA,AMASS,
     1RIN,IR,IRR,IN,IQ,N5,N6
      COMMON Z,Z1,Y,RADM,NREGS
      COMMON NX
      COMMON EPS1,EPS2,NINV1,NINV2

      DIMENSION G1(3),G2(3),G3(3),G4(3),G5(3),T(4)
      DIMENSION ARG(6),BSI0(6),BSI1(6),BSK0(6),BSK1(6),BSK2(6),BSK3(6)
     1SK4(6),BSK5(6),BSI2(6),BSI3(6),BSI4(6),BSI5(6)
      DIMENSION AA(1),BB(1),CC(1),D(1),E(1),F(1),G(1),H(1),W(1),X(1),
     1 SS(1),UU(1),P2(1),P3(1)
      COMMON ARG,BSI0,BSI1,BSK0,BSK1,BSK2,BSK3,BSK4,BSK5,BSI2,BSI3,BSI
     1BSI5

      CALCULATION OF GAMMA COEFFICIENTS
      NITER=0
      T(1)=1.0
      T(2)=0.0
      T(3)=1.0
      T(4)=11.76+55.44*Y
      BB=T(4)
    2 T(1)=T(2)
      T(2)=T(3)
      T(3)=T(4)
      T(4)=11.76*T(3)+9.24*Y*T(3)-15.4*T(2)-47.6*Y*T(2)+46.2*Y*T(1)
      T(1)=T(4)/T(3)
      CC=1.0-T(1)/BB
      D=ABSF(CC)
      BB=T(1)
      NITER=NITER+1
      IF(D-EPS1)3,3,55
   55 IF(NITER-50)2,2,56
   56 WRITE OUTPUT TAPE N6,57
   57 FORMAT(47H0 CONVERGENCE NOT ACHIEVED AFTER 50 ITERATIONS   //)
      WRITE OUTPUT TAPE N6,58,D,EPS1
   58 FORMAT(5H  D=  E12.5 ,12H WHILE EPS1= E12.5//)
    3 D=T(1)-11.76-9.24*Y
      E=15.4+47.6*Y+D*T(1)
      F=D*D-4.0*E
      G=SQRTF(F)
      H=G-D
      T(2)=0.5*H
      T(3)=E/T(2)
      DO 50 I=1,3
      UU=T(I)
      T(I)=SQRTF(UU)
      E=UU*UU
```

```
MATRIX

      F=E*T(I)
      H=UU*T(I)
      AA=-Y/T(I)
      BB=0.0324675324*E-0.381818182*UU-0.3*UU*Y+1.54545454*Y
      CC=-0.041322314*F+0.381818182*Y*H+0.431337856*H-1.466942148*Y*
      W=0.03047744*E*UU-0.31045134*E-0.28161156*Y*E+1.307544412*Y*UU
      X=-0.0138533847*E*H+0.1411142654*F+0.12800527652*Y*F-0.4579743(
     1H
      G1(I)=AA
      G2(I)=BB
      G3(I)=CC
      G4(I)=W
      G5(I)=X
   50 CONTINUE

      ARGUMENTS OF BESSEL FUNCTIONS
      ARG(1)=RADM*T(1)
      ARG(2)=RADM*T(2)
      ARG(3)=RADM*T(3)
      ARG(4)=3.92101531*RADM
      ARG(5)=1.43937374*RADM
      ARG(6)=3.3166247905*RADM

      DO 1000 I=1,6
      BSI0(I)=BI0F(ARG(I))
      BSI1(I)=BI1F(ARG(I))
      BSK0(I)=BK0F(ARG(I))
      BSK1(I)=BK1F(ARG(I))
      BSK2(I)=BSK0(I)+2.0*BSK1(I)/ARG(I)
      BSK3(I)=BSK1(I)+4.0*BSK2(I)/ARG(I)
      BSK4(I)=BSK2(I)+6.0*BSK3(I)/ARG(I)
      BSK5(I)=BSK3(I)+8.0*BSK4(I)/ARG(I)
      AA=0.01*BSI0(I)
      BB=0.0
      SS=15.0
      DO 30 J=1,11
      CC=2.0*SS*AA/ARG(I)+BB
      BB=AA
      AA=CC
   30 SS=SS-1.0
      P2=AA
      P3=BB
      DO 31 J=1,2
      CC=2.0*SS*AA/ARG(I)+BB
      BB=AA
      AA=CC
   31 SS=SS-1.0
      CC=4.0*AA/ARG(I)+BB
      CC=BSI1(I)/CC
      BSI2(I)=AA*CC
      BSI3(I)=BB*CC
      BSI4(I)=P2*CC
      BSI5(I)=P3*CC
 1000 CONTINUE

      MATRIX ASSEMBLY
```

```
 MATRIX

        DO  500  I=1,12
        DO  500  J=1,12
D  500  Z(I,J)=0.0
D       Z(1,1)=BSI0(1)
D       Z(1,2)=BSK0(1)
D       Z(1,3)=BSI0(2)
D       Z(1,4)=BSK0(2)
D       Z(1,5)=BSI0(3)
D       Z(1,6)=BSK0(3)
D       Z(2,1)=G1(1)*BSI1(1)
D       Z(2,2)=-G1(1)*BSK1(1)
D       Z(2,3)=G1(2)*BSI1(2)
D       Z(2,4)=-G1(2)*BSK1(2)
D       Z(2,5)=G1(3)*BSI1(3)
D       Z(2,6)=-G1(3)*BSK1(3)
D       Z(3,1)=-0.5*G2(1)*BSI0(1)
D       Z(3,2)=-0.5*G2(1)*BSK0(1)
D       Z(3,3)=-0.5*G2(2)*BSI0(2)
D       Z(3,4)=-0.5*G2(2)*BSK0(2)
D       Z(3,5)=-0.5*G2(3)*BSI0(3)
D       Z(3,6)=-0.5*G2(3)*BSK0(3)
D       Z(3,7)=BSI0(4)
D       Z(3,8)=BSK0(4)
D       Z(3,9)=BSI0(5)
D       Z(3,10)=BSK0(5)
D       Z(4,1)=3.0*G2(1)*BSI2(1)
D       Z(4,2)=3.0*G2(1)*BSK2(1)
D       Z(4,3)=3.0*G2(2)*BSI2(2)
D       Z(4,4)=3.0*G2(2)*BSK2(2)
D       Z(4,5)=3.0*G2(3)*BSI2(3)
D       Z(4,6)=3.0*G2(3)*BSK2(3)
D       Z(4,7)=2.0*BSI2(4)
D       Z(4,8)=2.0*BSK2(4)
D       Z(4,9)=2.0*BSI2(5)
D       Z(4,10)=2.0*BSK2(5)
D       Z(5,1)=-1.5*G3(1)*BSI1(1)
D       Z(5,2)=1.5*G3(1)*BSK1(1)
D       Z(5,3)=-1.5*G3(2)*BSI1(2)
D       Z(5,4)=1.5*G3(2)*BSK1(2)
D       Z(5,5)=-1.5*G3(3)*BSI1(3)
D       Z(5,6)=1.5*G3(3)*BSK1(3)
D       Z(5,7)=-1.252814036*BSI1(4)
D       Z(5,8)=1.252814036*BSK1(4)
D       Z(5,9)=-3.473732958*BSI1(5)
D       Z(5,10)=3.473732958*BSK1(5)
D       Z(6,1)=15.0*G3(1)*BSI3(1)
D       Z(6,2)=-15.0*G3(1)*BSK3(1)
D       Z(6,3)=15.0*G3(2)*BSI3(2)
D       Z(6,4)=-15.0*G3(2)*BSK3(2)
D       Z(6,5)=15.0*G3(3)*BSI3(3)
D       Z(6,6)=-15.0*G3(3)*BSK3(3)
D       Z(6,7)=-7.51688421*BSI3(4)
D       Z(6,8)=7.51688421*BSK3(4)
D       Z(6,9)=-20.84239775*BSI3(5)
D       Z(6,10)=20.84239775*BSK3(5)
D       Z(7,1)=0.375*G4(1)*BSI0(1)
```

MATRIX

```
D         Z(7,2)=0.375*G4(1)*BSK0(1)
D         Z(7,3)=0.375*G4(2)*BSI0(2)
D         Z(7,4)=0.375*G4(2)*BSK0(2)
D         Z(7,5)=0.375*G4(3)*BSI0(3)
D         Z(7,6)=0.375*G4(3)*BSK0(3)
D         Z(7,7)=0.233553315*BSI0(4)
D         Z(7,8)=0.233553315*BSK0(4)
D         Z(7,9)=-0.99112907*BSI0(5)
D         Z(7,10)=-0.99112907*BSK0(5)
D         Z(7,11)=BSI0(6)
D         Z(7,12)=BSK0(6)
D         Z(8,1)=-7.5*G4(1)*BSI2(1)
D         Z(8,2)=-7.5*G4(1)*BSK2(1)
D         Z(8,3)=-7.5*G4(2)*BSI2(2)
D         Z(8,4)=-7.5*G4(2)*BSK2(2)
D         Z(8,5)=-7.5*G4(3)*BSI2(3)
D         Z(8,6)=-7.5*G4(3)*BSK2(3)
D         Z(8,7)=-2.80263978*BSI2(4)
D         Z(8,8)=-2.80263978*BSK2(4)
D         Z(8,9)=11.8935439*BSI2(5)
D         Z(8,10)=11.8935439*BSK2(5)
D         Z(8,11)=12.0*BSI2(6)
D         Z(8,12)=12.0*BSK2(6)
D         Z(9,2)=105.0*G4(1)*BSK4(1)
D         Z(9,3)=105.0*G4(2)*BSI4(2)
D         Z(9,4)=105.0*G4(2)*BSK4(2)
D         Z(9,5)=105.0*G4(3)*BSI4(3)
D         Z(9,6)=105.0*G4(3)*BSK4(3)
D         Z(9,7)=-39.236957*BSI4(4)
D         Z(9,8)=-39.236957*BSK4(4)
D         Z(9,9)=166.509685*BSI4(5)
D         Z(9,10)=166.509685*BSK4(5)
D         Z(9,11)=24.0*BSI4(6)
D         Z(9,1)=105.0*G4(1)*BSI4(1)
D         Z(9,12)=24.0*BSK4(6)
D         Z(10,1)=1.875*G5(1)*BSI1(1)
D         Z(10,2)=-1.875*G5(1)*BSK1(1)
D         Z(10,3)=1.875*G5(2)*BSI1(2)
D         Z(10,4)=-1.875*G5(2)*BSK1(2)
D         Z(10,5)=1.875*G5(3)*BSI1(3)
D         Z(10,6)=-1.875*G5(3)*BSK1(3)
D         Z(10,7)=-1.7794919942*BSI1(4)
D         Z(10,8)=1.7794919942*BSK1(4)
D         Z(10,9)=2.723518954*BSI1(5)
D         Z(10,10)=-2.723518954*BSK1(5)
D         Z(10,11)=-2.7136021*BSI1(6)
D         Z(10,12)=2.7136021*BSK1(6)
D         Z(11,1)=-52.5*G5(1)*BSI3(1)
D         Z(11,2)=52.5*G5(1)*BSK3(1)
D         Z(11,3)=-52.5*G5(2)*BSI3(2)
D         Z(11,4)=52.5*G5(2)*BSK3(2)
D         Z(11,5)=-52.5*G5(3)*BSI3(3)
D         Z(11,6)=52.5*G5(3)*BSK3(3)
D         Z(11,7)=21.35390393*BSI3(4)
D         Z(11,8)=-21.35390393*BSK3(4)
D         Z(11,9)=-32.6822274*BSI3(5)
```

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 MATRIX

D       Z(11,10)=30.6822274*BSK3(5)
D       Z(11,11)=-97.689675*BSI3(6)
D       Z(11,12)=97.689675*BSK3(6)
D       Z(12,1)=945.0*G5(1)*BSI5(1)
D       Z(12,2)=-945.0*G5(1)*BSK5(1)
D       Z(12,3)=945.0*G5(2)*BSI5(2)
D       Z(12,4)=-945.0*G5(2)*BSK5(2)
D       Z(12,5)=945.0*G5(3)*BSI5(3)
D       Z(12,6)=-945.0*G5(3)*BSK5(3)
D       Z(12,7)=640.61711781*BSI5(4)
D       Z(12,8)=-640.61711781*BSK5(4)
D       Z(12,9)=-980.4668234*BSI5(5)
D       Z(12,10)=980.4668234*BSK5(5)
D       Z(12,11)=-325.63225*BSI5(6)
D       Z(12,12)=325.63225*BSK5(6)
C
        IF(NX)7000,7001,7000
 7000   CONTINUE
        WRITE OUTPUT TAPE N6,3007,T(1),T(2),T(3)
 3007   FORMAT(11H1 GAMMA(1)=E12.5/11H  GAMMA(2)=E12.5/11H  GAMMA(3)=
       1 E12.5//)
        WRITE OUTPUT TAPE N6,3001
 3001   FORMAT(21H0     G  COEFFICIENTS // 5X,2HG1,10X,2HG2,10X,2HG3,10X
       1 2HG4,10X,2HG5//)
        WRITE OUTPUT TAPE N6,3000,(G1(I),G2(I),G3(I),G4(I),G5(I),I=1,3)
 3000   FORMAT(5E12.5)
        WRITE OUTPUT TAPE N6,3002,RADM,Y
 3002   FORMAT(8H0 RADM= E12.5,5X,8H ALPHA= E12.5)
        WRITE OUTPUT TAPE N6,3003,(ARG(I),I=1,6)
 3003   FORMAT(32H0 ARGUMENTS OF BESSEL FUNCTIONS // 6E12.5 ///)
        WRITE OUTPUT TAPE N6,3004
 3004   FORMAT(19H0 BESSEL FUNCTIONS // 5X,2HI0,10X,2HI1,10X,2HI2,10X,
       1 2HI3,10X,2HI4,10X,2HI5/)
        WRITE OUTPUT TAPEN6,3005,(BSI0(I),BSI1(I),BSI2(I),BSI3(I),BSI4(
       1 BSI5(I),I=1,6)
 3005   FORMAT(6E12.5)
        WRITE OUTPUT TAPE N6,3006
 3006   FORMAT(2H0 3X,2HK0,10X,2HK1,10X,2HK2,10X,2HK3,
       1 10X,2HK4,10X,2HK5/)
        WRITE OUTPUT TAPEN6,3005,(BSK0(I),BSK1(I),BSK2(I),BSK3(I),BSK4(
       1 BSK5(I),I=1,6)
C
        M=12
        N=12
 2000   FORMAT (6E20.5)
 2001   FORMAT(1H1//)
 2002   FORMAT(1H0//)
        WRITE OUTPUT TAPE N6,2001
        IF(M-6)10,10,11
   10   N1=M
        WRITE OUTPUT TAPE N6,2000,((Z(I,J),I=1,N1),J=1,N)
D       RETURN
   11   N1=6
        WRITE OUTPUT TAPE N6,2000,((Z(I,J),J=1,N1),I=1,N)
        WRITE OUTPUT TAPE N6,2002
        WRITE OUTPUT TAPE N6,2000,((Z(I,J),J=7,M),I=1,N)


 MATRIX

C
 7001   CONTINUE
D       RETURN
        END(1,0,0,0,0,0,1,0,0,0,0,0,0,0,0)
```

SIMH

```
D        SUBROUTINE SIMH(A,AT,N,ITER)
C
C        MATRIX INVERSION SUBROUTINE
C
C        A(I,J)           MATRIX TO BE INVERTED
C        AT(I,J)          INVERTED MATRIX
C        N                MATRIX ORDER
C        ITER             NO. OF ITERATIONS IN THE INVERSION
C
D        DIMENSION A(12,12),AT(12,12)
       3 DO 4  I=1,N
         DO 4  J=1,N
D      4 AT(I,J)=A(J,I)
D        DET=1.
         DO 800IL=1,ITER
         DO 800I=1,N
D        C=0.
         DO 100K=1,N
D    100 C=C+A(I,K)*AT(K,I)
D        IF(C)200,555,200
D    200 DET=DET*C
         DO 300J=1,N
D    300 AT(J,I)=AT(J,I)/C
         DO 700J=1,N
         IF(J-I)400,700,400
D    400 H=0.
         DO 500K=1,N
D    500 H=H+A(I,K)*AT(K,J)
         DO 600K=1,N
D    600 AT(K,J)=AT(K,J)-H*AT(K,I)
     700 CONTINUE
     800 CONTINUE
D        RETURN
     555 WRITE OUTPUT TAPE 6,1000
    1000 FORMAT(25H1 MATRIX ILL CONDITIONED //)
         CALL EXIT
         END(1,0,0,0,0,0,1,0,0,0,0,0,0,0,0)
```

```
AIRGAP

      SUBROUTINE AIRGAP
      DIMENSION        A(12,12,10),    B(12,12,10),    U(12,6,10),
     1 VAN(12,12),   U1(12,6),        RESP(12,12),    V(12,10),
     2 RV(6),         V1(12),          S(12,10),       C(12,10),
     3 RAD(10),FMP(10),SOURCE(10),ALPHA(10),AMASS(10),APRIN(10)
      DIMENSION Z(12,12),Z1(12,12)
      DIMENSION Y(1),RADM(1)
      DIMENSION EPS1(1),EPS2(1)
      COMMON A,B,U,VAN,U1,RESP,V,RV,V1,S,C,RAD,FMP,SOURCE,ALPHA,AMASS,
     1RIN,IR,IRR,IN,IQ,N5,N6
      COMMON Z,Z1,Y,RADM,NREGS
      COMMON NX
      COMMON EPS1,EPS2,NINV1,NINV2
      DIMENSION AA(1),BB(1),PI(1),CC(1),D(1),E(1),G(1),H(1),UU(1),VV(1
     1 F(1),B3(1),ZZ(1)
      PI=3.1415926535
      AA=RAD(IN)**3
      BB=RAD(IN)**5
      DO 10 I=1,12
      DO 10 J=1,12
   10 Z(I,J)=0.0
      Z(1,2)=RAD(IN)
      Z(2,5)=RAD(IN)
      Z(3,10)=RAD(IN)
      Z(4,2)=-12.0*AA
      Z(4,5)=2.0*AA
      Z(4,6)=AA
      Z(5,2)=-12.0*AA
      Z(5,5)=-28.0*AA
      Z(5,10)=12.0*AA
      Z(5,11)=AA
      Z(6,2)=1296.0*BB
      Z(6,5)=-336.0*BB
      Z(6,6)=-168.0*BB
      Z(6,10)=48.0*BB
      Z(6,11)=6.0*BB
      Z(6,12)=BB
      Z(7,1)=-PI/4.0
      Z(7,2)=PI/2.0
      Z(7,3)=0.15625*PI
      Z(7,4)=-0.078125*PI
      Z(7,7)=0.03515625*PI
      Z(7,8)=-0.00390625*PI
      Z(7,9)=0.00048828125*PI
      Z(8,1)=-0.09375*PI
      Z(8,3)=-0.8203125*PI
      Z(8,4)=0.01953125*PI
      Z(8,5)=0.5*PI
      Z(8,7)=0.507568359*PI
      Z(8,8)=-0.041748046*PI
      Z(8,9)=-0.000274658*PI
      Z(9,1)=-0.05859375*PI
      Z(9,3)=-0.34790039*PI
      Z(9,4)=0.00305175781*PI
```

```
  AIRGAP

D        Z(9,7)=-1.289520263*PI
D        Z(9,8)=0.015106201*PI
D        Z(9,9)=0.000034332275*PI
D        Z(9,10)=0.5*PI
D        Z(10,1)=0.9375*PI
D        Z(10,3)=-1.171875*PI
D        Z(10,4)=-1.7578125*PI
D        Z(10,6)=0.5*PI
D        Z(10,7)=0.197753906*PI
D        Z(10,8)=0.06591796*PI
D        Z(10,9)=-0.04119873*PI
D        Z(11,1)=1.640625*PI
D        Z(11,3)=5.639648437*PI
D        Z(11,4)=-0.769042968*PI
D        Z(11,7)=-10.03601074*PI
D        Z(11,8)=-2.514770507*PI
D        Z(11,9)=0.028839111*PI
D        Z(11,11)=0.5*PI
D        Z(12,1)=-29.53125*PI
D        Z(12,3)=46.14257812*PI
D        Z(12,4)=38.45214843*PI
D        Z(12,7)=-18.68774414*PI
D        Z(12,8)=-3.46069335*PI
D        Z(12,9)=-3.02810668*PI
D        Z(12,12)=0.5*PI
         DO  11  I=1,12
         DO  11  J=1,12
D    11  B(I,J,IN)=Z(I,J)
C
D        SOURCE(IN)=0.0
D        AA=RAD(IN)/RAD(IN+1)
D        BB=1.0-AA*AA
D        BB=SQRTF(BB)
D        CC=ATANF(AA/BB)
D        D=AA*BB
D        E=CC-D
D        G=AA*AA
D        H=G*AA
D        UU=G*G
D        VV=UU*AA
D        F=RAD(IN+1)**3
D        B3=BB**3
D        ZZ=RAD(IN+1)**5
D        EE=1.-(2./3.)*G
D        DD=(1.-G)**(3./2.)
D        EEE=CC+D
D        A1=1.-1./G
         DO  22  I=1,12
         DO  22  J=1,12
D    22  Z(I,J)=0.0
D        Z(1,2)=RAD(IN+1)
D        Z(2,5)=RAD(IN+1)
D        Z(3,10)=RAD(IN+1)
D        Z(4,6)=F
D        Z(4,2)=-12.*F
D        Z(4,5)=2.*F
```

AIRGAP

```
Z(5,11)=F
Z(5,2)=-12.*F
Z(5,5)=-28.*F
Z(5,10)=12.*F
Z(6,12)=ZZ
Z(6,2)=1296.*ZZ
Z(6,5)=-336.*ZZ
Z(6,10)=48.*ZZ
Z(6,6)=-168.*ZZ
Z(6,11)=6.*ZZ
Z(7,1)=-PI/4.
Z(7,2)=EEE/AA
Z(7,3)=(5.*PI)/32.
Z(7,4)=-((15.*PI)/64.)*EE
Z(7,6)=(7./45.)*DD
Z(7,7)=(9.*PI)/256.
Z(7,8)=-((3.*PI)/256.)*EE
Z(7,9)=-(PI/2048.)*(15.-40.*G+24.*UU)
Z(7,11)=(11./1260.)*DD
Z(7,12)=(11./4200.)*DD*(1.-(8./3.)*G)
Z(8,1)=-(3.*PI)/32.
Z(8,3)=-(105.*PI)/128.
Z(8,4)=((15.*PI)/256.)*EE
Z(8,5)=EEE/AA
Z(8,6)=-(1./15.)*DD
Z(8,7)=(2079.*PI)/4096.
Z(8,8)=-((513.*PI)/4096.)*EE
Z(8,9)=((9.*PI)/32768.)*(15.-40.*G+24.*UU)
Z(8,11)=(11./210.)*DD
Z(8,12)=-(11./6300.)*DD*(1.-(8./3.)*G)
Z(9,1)=-(15.*PI)/256.
Z(9,3)=-(1425.*PI)/4096.
Z(9,4)=((75.*PI)/8192.)*EE
Z(9,7)=-(42255.*PI)/32768.
Z(9,8)=((1485.*PI)/32768.)*EE
Z(9,9)=-((7.*PI)/262144.)*(15.-40.*G+24.*UU)
Z(9,10)=EEE/AA
Z(9,11)=-(1./28.)*DD
Z(9,12)=(1./2520.)*DD*(1.-(8./3.)*G)
Z(10,1)=(15.*PI)/16.
Z(10,2)=(12.*A1*E)/AA
Z(10,3)=-(75.*PI)/64.
Z(10,4)=((375.*PI)/128.)*(1./3.-(14./15.)*G)
Z(10,5)=-(2.*A1*E)/AA
Z(10,6)=(CC-D*(1.+(4./3.)*G-(10./3.)*UU))/H
Z(10,7)=(405.*PI)/2048.
Z(10,8)=-((225.*PI)/2048.)*(1./3.-(14./15.)*G)
Z(10,9)=(15.*PI/16384.)*(35.-392.*G+312.*UU)
Z(10,12)=-(11./945.)*BB*(1.-(17.*G)+16.*UU)
Z(11,1)=(105.*PI)/64.
Z(11,2)=(12.*A1*E)/AA
Z(11,3)=(5775.*PI)/1024.
Z(11,4)=((175.*PI)/2048.)*(5.-14.*G)
Z(11,5)=(28.*A1*E)/AA
Z(11,7)=-(82215.*PI)/8192.
Z(11,8)=((2289.*PI)/8192.)*(5.-14.*G)
```

```
AIRGAP
      Z(11,9)=-((21.*PI)/65536.)*(35.-390.*G+312.*UU)
      Z(11,10)=-(12.*A1*E)/AA
      Z(11,11)=(CC-D*(1.+(4./3.)*G-(10./3.)*UU))/H
      Z(11,12)=(1./135.)*BB*(1.-17.*G+16.*UU)
      Z(12,1)=-(945.*PI)/32.
      Z(12,2)=  (648./VV)*((2.-3.*G+UU)*CC-D*(2.-(5./3.)*G-(1./3.)*UU
      Z(12,3)=(23625.*PI)/512.
      Z(12,4)=-((1575.*PI)/1024.)*(21.-46.*G)
      Z(12,5)=-(168./VV)*((2.-3.*G+UU)*CC-D*(2.-(5./3.)*G-(1./3.)*UU
      Z(12,6)=  -(56./VV)*(3.*(1.-G)*CC-D*(3.-G-2.*H*H))
      Z(12,7)=-(76545.*PI)/4096.
      Z(12,8)=((567.*PI)/4096.)*(21.-46.*G)
      Z(12,9)=-((567.*PI)/32768.)*(63.-552.*G+664.*UU)
      Z(12,10)=  (24./VV)*((2.-3.*G+UU)*CC-D*(2.-(5./3.)*G-(1./3.)*UU
      Z(12,11)=   (2./VV)*(3.*(1.-G)*CC-D*(3.-G-2.*H*H))
      Z(12,12)=(CC-D*(1.+(2./3.)*G+(2./15.)*UU+(86./15.)*H*H-(128./1
     1*UU*UU))/VV
8200  FORMAT(//(6E20.5))
8201  FORMAT(1H1,52X,17H AIRGAP  MATRICES////19H          INNER  RADIUS)
8301  FORMAT(///19H        OUTER  RADIUS)
      IF(NX) 8202,8202,8203
8203  WRITE OUTPUT TAPE N6,8201
      WRITE OUTPUT TAPE N6,8200,((B(I,J,IN),J=1,6),I=1,12)
      WRITE OUTPUT TAPE N6,8200,((B(I,J,IN),J=7,12),I=1,12)
      WRITE OUTPUT TAPE N6,8301
      WRITE OUTPUT TAPE N6,8200,((Z(I,J),J=1,6),I=1,12)
      WRITE OUTPUT TAPE N6,8200,((Z(I,J),J=7,12),I=1,12)
8202  CALL SIMH(Z,Z1,12,NINV1)
      DO 23 I=1,12
      DO 23 J=1,12
   23 A(I,J,IN+1)=Z1(I,J)
      RETURN
      END(1,0,0,0,0,0,1,0,0,0,0,0,0,0,0)
```

PRINT

```
C
D     SUBROUTINE PRINT
C
C     FLUXM(IN)       MEAN FLUX REGION IN
C     VOLUME(IN)      VOLUME REGION IN
C     S(I,IN)         SOURCE VECTOR REGION IN
C     ASFLUX          ASYMPTOTIC FLUX
C     ASCURR          ASYMPTOTIC CURRENT
C
C
D     DIMENSION       A(12,12,10),    B(12,12,10),    U(12,6,10),
     1 VAN(12,12),    U1(12,6),       RESP(12,12),    V(12,10),
     2 RV(6),         V1(12),         S(12,10),       C(12,10),
     3 RAD(10),FMP(10),SOURCE(10),ALPHA(10),AMASS(10),APRIN(10)
D     DIMENSION Z(12,12),Z1(12,12)
D     DIMENSION Y(1),RADM(1)
D     DIMENSION EPS1(1),EPS2(1)
D     COMMON A,B,U,VAN,U1,RESP,V,RV,V1,S,C,RAD,FMP,SOURCE,ALPHA,AMASS,
     1RIN,IR,IRR,IN,IQ,N5,N6
D     COMMON Z,Z1,Y,RADM,NREGS
      COMMON NX
D     COMMON EPS1,EPS2,NINV1,NINV2
C     DIMENSION DELTA(1),FLUX(1),RPRIN(1)
D     DIMENSION XN(1),UN(1),VN(1),YN(1),WN(1),ASFLUX(1),ASCURR(1),
     1 FLUXM(10)
D     DIMENSION ARG(6),BSI0(6),BSI1(6),BSK0(6),BSK1(6),BSK2(6),BSK3(6)
     1SK4(6),BSK5(6),BSI2(6),BSI3(6),BSI4(6),BSI5(6)
D     COMMON ARG,BSI0,BSI1,BSK0,BSK1,BSK2,BSK3,BSK4,BSK5,BSI2,BSI3,BSI
     1BSI5
      DIMENSION VOLUME(10)
      NX=0
    2 FORMAT (F12.5,E15.8,3X,E15.8,3X,E15.8)
      WRITE OUTPUT TAPE N6,1
    1 FORMAT (10H1  RADIUS 8X,4HFLUX,8X,15HASYMPTOTIC FLUX,20H   ASYMPT
     1IC CURRENT //)
D     RPRIN=0.0
D     FLUX=C(1,1)+C(3,1)+C(5,1)+S(1,1)
      WRITE OUTPUT TAPE N6,2,RPRIN,FLUX
      DO 10 IN=1,NREGS
      IF(FMP(IN))20,21,21
   20 WRITE OUTPUT TAPE N6,22
   22 FORMAT (11H0  AIR GAP //)
      RPRIN=RAD(IN)
      GO TO 10
   21 N=APRIN(IN)+2.01
D     DELTA=(RAD(IN)-RPRIN)/(APRIN(IN)+1.0)
D     XN=0.0
D     YN=2.0*FMP(1)/(RAD(1)*RAD(1))
      DO 11 IP=1,N
      IF(RPRIN)11,11,12
D  12 RADM=RPRIN/FMP(IN)
D     Y=ALPHA(IN)
D     CALL MATRIX
C
      IF(IP-1)51,50,51
   51 CONTINUE
```

```
 PRINT
C       MATRIX MULTIPLICATION AND ADDITION  V1=A*C+S
        DO 13 I=1,12
D       V1(I)=0.0
        DO 13 IS=1,12
D   13  V1(I)=V1(I)+Z(I,IS)*C(IS,IN)
        DO 14 I=1,12
D   14  V1(I)=V1(I)+S(I,IN)
D       FLUX=V1(1)
D       ASFLUX=Z(1,5)*C(5,IN)+Z(1,6)*C(6,IN)+S(1,IN)
D       ASCURR=Z(2,5)*C(5,IN)+Z(2,6)*C(6,IN)
        WRITE OUTPUT TAPE N6,2,RPRIN,FLUX,ASFLUX,ASCURR
        IF(IP-N)52,53,52
    52  CONTINUE
D   11  RPRIN=RPRIN+DELTA
D       RPRIN=RPRIN-DELTA
    10  CONTINUE
        IF(IQ)1002,1003,1002
  1002  WRITE OUTPUT TAPE N6,1000
  1000  FORMAT(21H1 CONTROL ON ACCURACY/29H  FLUX AT REFLECTIVE BOUNDA
       1 / 45H  THE COMPONENTS MARKED WITH * SHOULD VANISH //)
        WRITE OUTPUT TAPE N6,1001,(V1(I),I=1,12)
  1001  FORMAT(E13.5/E13.5,2H */E13.5/E13.5/E13.5,2H */E13.5,2H */
       1 E13.5/E13.5/E13.5/E13.5,2H */E13.5,2H */E13.5,2H */)
  1003  CONTINUE
D       RETURN
D   50  XN=C(1,IN)*BSI1(1)-C(2,IN)*BSK1(1)
D       XN=RPRIN*XN*RADM/ARG(1)
D       UN=C(3,IN)*BSI1(2)-C(4,IN)*BSK1(2)
D       UN=RPRIN*UN*RADM/ARG(2)
D       VN=C(5,IN)*BSI1(3)-C(6,IN)*BSK1(3)
D       VN=RPRIN*VN*RADM/ARG(3)
D       XN=XN+UN+VN
D       YN=RAD(IN)**2-RPRIN**2
D       YN=2.0*FMP(IN)/YN
        GO TO 51
D   53  WN=C(1,IN)*BSI1(1)-C(2,IN)*BSK1(1)
D       WN=RPRIN*WN*RADM/ARG(1)
D       UN=C(3,IN)*BSI1(2)-C(4,IN)*BSK1(2)
D       UN=RPRIN*UN*RADM/ARG(2)
D       VN=C(5,IN)*BSI1(3)-C(6,IN)*BSK1(3)
D       VN=RPRIN*VN*RADM/ARG(3)
D       WN=WN+UN+VN-XN
D       FLUXM(IN)=YN*WN+S(1,IN)
        VOLUME(IN)=3.1415926535*(RAD(IN)*RAD(IN)-RAD(IN-1)*RAD(IN-1))
        WRITE OUTPUT TAPE N6,60,IN,FLUXM(IN),VOLUME(IN)
    60  FORMAT (8H REGION I3,13H MEAN FLUX = E12.5,4X,9HVOLUME = E12.5
        GO TO 52
        END(1,0,0,0,0,0,1,0,0,0,0,0,0,0,0)
```

The double precision Bessel
function subroutines are not
included in this listing.

3 REGIONS

SAMPLE OUTPUT

```
                  REGION =    1
            OUTER RADIUS =  0.12700E 01
          MEAN FREE PATH =  0.13826E 01
                  SOURCE =  0.
NO.ABSORPTIONS/COLLISION =  0.46710E-00


                  REGION =    2
            OUTER RADIUS =  0.34925E 01
          MEAN FREE PATH = -0.10000E 01
                  SOURCE =  0.
NO.ABSORPTIONS/COLLISION =  0.10000E 01


                  REGION =    3
            OUTER RADIUS =  0.11285E 02
          MEAN FREE PATH =  0.24612E 01
                  SOURCE =  0.10000E 01
NO.ABSORPTIONS/COLLISION =  0.10880E-02
```

```
   RADIUS            FLUX          ASYMPTOTIC FLUX  ASYMPTOTIC CURRENT

     0.       0.15428335E 03
     0.25400  0.15602420E 03      0.20046557E 03    -0.85674568E 01
     0.50800  0.16160909E 03      0.20504069E 03    -0.17330567E 02
     0.76200  0.17226632E 03      0.21278194E 03    -0.26489948E 02
     1.01600  0.19057240E 03      0.22386651E 03    -0.36256278E 02
     1.27000  0.22147449E 03      0.23854876E 03    -0.46855665E 02
REGION    1 MEAN FLUX =  0.18320E 03     VOLUME =  0.50671E 01

   AIR GAP

     3.49250  0.26184095E 03      0.26322768E 03    -0.14287844E 02
     4.79128  0.28099620E 03      0.28162068E 03    -0.94277818E 01
     6.09007  0.29374038E 03      0.29398270E 03    -0.64048030E 01
     7.38885  0.30230287E 03      0.30232582E 03    -0.42506678E 01
     8.68763  0.30782378E 03      0.30767512E 03    -0.25756345E 01
     9.98642  0.31095890E 03      0.31062385E 03    -0.11923563E 01
    11.28520  0.31207167E 03      0.31154570E 03     0.54373478E-03
REGION    3 MEAN FLUX =  0.30154E 03     VOLUME =  0.36178E 03
```

```
  CONTROL ON ACCURACY
  FLUX AT REFLECTIVE BOUNDARY
  THE COMPONENTS MARKED WITH * SHOULD VANISH

  0.31207E 03
 -0.23668E-10  *
  0.40019E-00
 -0.32127E 01
  0.59039E-06  *
  0.31665E-05  *
  0.11746E-01
 -0.30952E-00
 -0.42362E 02
  0.79679E-06  *
 -0.84697E-05  *
 -0.17324E-03  *
```

To disseminate knowledge is to disseminate prosperity — I mean general prosperity and not individual riches — and with prosperity disappears the greater part of the evil which is our heritage from darker times.

Alfred Nobel

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